

II Etude dynamique:

II.1 bilan des forces agissant sur  $S_4$

\* Action de l'arbre 3 et 5:

$$\left\{ \mathcal{C}(S_3 \rightarrow S_4) \right\}_C = \left\{ \begin{array}{l} F_x \quad C_m \\ F_y \quad 0 \\ F_z \quad 0 \end{array} \right\}_C \quad \begin{array}{l} \text{exprimé dans} \\ \text{la base } B(\vec{x}, \vec{y}, \vec{z}) \end{array}$$

$\uparrow$   $R_{5 \rightarrow 4}$        $\nwarrow$   $m_{5-4}(C)$

\* Action de la pompe

$$\left\{ \mathcal{C}(P \rightarrow S_4) \right\} = \left\{ \begin{array}{l} 0 \quad C_r \\ 0 \quad 0 \\ 0 \quad 0 \end{array} \right\} \quad \begin{array}{l} \text{exprimé dans la} \\ \text{base } B(\vec{x}, \vec{y}, \vec{z}) \end{array}$$

\* Action de la liaison pivot rotule en O

$$\left\{ \mathcal{C}(S_0 \rightarrow S_4) \right\}_O = \left\{ \begin{array}{l} x_0 \quad 0 \\ y_0 \quad 0 \\ z_0 \quad 0 \end{array} \right\} \quad \begin{array}{l} \text{exprimé dans} \\ B_4(\vec{x}_4, \vec{y}_4, \vec{z}_4) \end{array}$$

\* Action de la liaison linéaire anulaire en B.

$$\left\{ \mathcal{C}(S_0 \rightarrow S_4) \right\}_B = \left\{ \begin{array}{l} 0 \quad 0 \\ y_B \quad 0 \\ z_B \quad 0 \end{array} \right\} \quad \begin{array}{l} \text{exprimé dans} \\ B_4(\vec{x}_4, \vec{y}_4, \vec{z}_4) \end{array}$$

\* Action de la pesanteur:

$$\left\{ \mathcal{C}(g \rightarrow S_{4a}) \right\}_C = \left\{ \begin{array}{l} -m_{4a} g \vec{y} \\ \vec{0} \end{array} \right\}_C$$

$$\left\{ \mathcal{C}(g \rightarrow S_{4b}) \right\}_{G_1} = \left\{ \begin{array}{l} -m_{4b} g \vec{y} \\ \vec{0} \end{array} \right\}_{G_1}$$

II.2 Principe fondamental de la dynamique:

$$\left\{ \mathcal{D}(S_4/R_0) \right\} = \left\{ \mathcal{C}(\vec{S}_4 \rightarrow S_4) \right\}$$

$$\vec{\delta}_0 (S_{ub}/R_0) = \frac{d}{dt} \left( \vec{v}_0 (S_{ub}/R_0) \right)$$

$$\vec{v}_0 (S_{ub}/R_0) = I (O, S_{ub}) \cdot \vec{\omega} (S_{ub}/R_0)$$

$$= \begin{bmatrix} A_b & -F_b & \cdot \\ -F_b & B_b & \cdot \\ \cdot & \cdot & C_b \end{bmatrix} \begin{Bmatrix} \dot{\theta} \\ 0 \\ 0 \end{Bmatrix}$$

$$= A_b \dot{\theta} \vec{x}_u - F_b \dot{\theta} \vec{y}_u$$

$$\Rightarrow \vec{\delta}_0 (S_{ub}/R_0) = A_b \ddot{\theta} \vec{x}_u - F_b \ddot{\theta} \vec{y}_u - \dot{\theta}^2 \vec{z}_u$$

$$\Rightarrow \vec{\delta}_0 (S_u/R_0) = [A_a + A_b] \ddot{\theta} \vec{x}_u - F_b \ddot{\theta} \vec{y}_u - \dot{\theta}^2 \vec{z}_u$$

II.3 théorème de la résultante dynamique:

$$- m_{4b} y_{G_4} \ddot{\theta}^2 \vec{y}_u + m_{4b} y_{G_4} \ddot{\theta} \vec{z}_u = (F_x + X_0) \vec{x}_u$$

$$+ (F_y - m_4 g) \vec{y} + F_z \vec{z} + (Y_0 + Y_B) \vec{y}_u$$

$$+ (Z_0 + Z_B) \vec{z}_u$$

$$\vec{y} = \cos \theta \vec{y}_u - \sin \theta \vec{z}_u ; \vec{z} = \sin \theta \vec{y}_u + \cos \theta \vec{z}_u$$

$$\Rightarrow - m_{4b} y_{G_4} \dot{\theta}^2 \vec{y}_u + m_{4b} y_{G_4} \ddot{\theta} \vec{z}_u = (F_x + X_0) \vec{x}_u$$

$$+ [(F_y - m_4 g) \cos \theta + Y_0 + Y_B] \vec{y}_u$$

$$+ [-(F_y - m_4 g) \sin \theta + Z_0 + Z_B] \vec{z}_u$$

$$\Rightarrow \textcircled{I} \begin{cases} F_x + X_0 = 0 \\ -m_{4b} y \ddot{\theta} = (F_y - m_{4b} g) \cos \theta + Y_0 + Y_B \\ m_{4b} y \ddot{\theta} = -(F_y - m_{4b} g) \sin \theta + Z_0 + Z_B \end{cases}$$

théorème du moment dynamique:

$$\begin{aligned} & [A_a + A_b] \ddot{\theta} \vec{x}_4 - F_b \ddot{\theta} \vec{y}_4 - F_b \dot{\theta}^2 \vec{z}_4 \\ &= (C_r + m_{4b} g y \sin \theta) \vec{x}_4 + [-C F_z \vec{y}_4 + (C F_y - m_{4b} g x_a) \vec{z}_4 \\ & \quad + b Y_B \vec{z}_4 - b Z_B \vec{y}_4] \\ &= (C_r + m_{4b} g y \sin \theta) \vec{x}_4 + [-C F_z \cos \theta - b Z_B] \vec{y}_4 \\ & \quad + [C F_z \sin \theta + b Y_B] \vec{z}_4 + (C F_y - m_{4b} g x_a) \sin \theta \vec{y}_4 \\ & \quad + (C F_y - m_{4b} g x_a) \cos \theta \vec{z}_4 \\ &= (C_r + m_{4b} g y \sin \theta) \vec{x}_4 + [-C F_z \cos \theta - b Z_B + C F_y \sin \theta \\ & \quad - m_{4b} g x_a \sin \theta] \vec{y}_4 + [C F_z \sin \theta + b Y_B + C F_y \cos \theta \\ & \quad - m_{4b} g x_a \cos \theta] \vec{z}_4 \end{aligned}$$

$$\Rightarrow \textcircled{II} \begin{cases} (A_a + A_b) \ddot{\theta} = C_r + m_{4b} g y \sin \theta \\ -F_b \ddot{\theta} = -C F_z \cos \theta - b Z_B + C F_y \sin \theta - m_{4b} g x_a \sin \theta \\ -F_b \dot{\theta}^2 = C F_z \sin \theta + b Y_B + C F_y \cos \theta - m_{4b} g x_a \cos \theta \end{cases}$$